9.7: 32, 36; Three More Problems

32. (a) The hollow ball is a spherical shell with outer radius 15 cm and inner radius 14.5 cm. If we center the ball at the origin of the coordinate system and use centimeters as the unit of measurement, then spherical coordinates conveniently describe the hollow ball as $14.5 \leq \rho \leq 15, 0 \leq \theta \leq 2\pi, 0 \leq \phi \leq \pi$.

(b) If we position the ball as in part (a), one possibility is to take the half of the ball that is above the xy-plane which is described by $14.5 \leq \rho \leq 15, 0 \leq \theta \leq 2\pi, 0 \leq \phi \leq \pi/2$.

36. We begin by finding the positions of Los Angeles and Montréal in spherical coordinates, using the method described in the exercise:

Montréal	Los Angeles
$\rho = 3960$ mi	$\rho = 3960$ mi
$\theta = 360^\circ - 73.60^\circ = 286.40^\circ$	$\theta = 360^\circ - 118.25^\circ = 241.75^\circ$
$\phi = 90^\circ - 45.50^\circ = 44.50^\circ$	$\phi = 90^\circ - 34.06^\circ = 55.94^\circ$

Now we change the above to Cartesian coordinates using $x = \rho\cos\theta\sin\phi$, $y = \rho\sin\theta\sin\phi$ and $z = \rho\cos\phi$ to get two position vectors of length 3960 mi (since both cities must lie on the surface of the Earth). In particular:

Montréal: $\langle 783.67, -2662.67, 2824.47\rangle$

Los Angeles: $\langle -1552.80, -2889.91, 2217.84\rangle$

To find the angle α between these two vectors we use the dot product:
$\langle 783.67, -2662.67, 2824.47\rangle \cdot \langle -1552.80, -2889.91, 2217.84\rangle = (3960)^2\cos\alpha \quad\Rightarrow\quad \cos\alpha \approx 0.8126 \quad\Rightarrow$
$\alpha \approx 0.6223$ rad. The great circle distance between the cities is $s = \rho\theta \approx 3960(0.6223) \approx 2464$ mi.